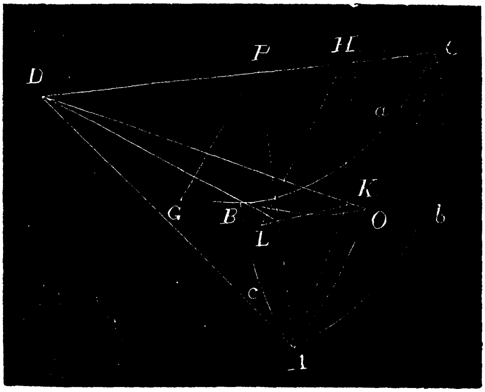
DEMONSTRATION OF A PROPOSITION (SEE QUERY, P. 55).

BY R. J. ADCOCK, MONMOUTH, ILL.

Let ABC represent the triangle. Draw AP perpendicular to CD, the radius of the sphere. In the plane DBC, perpendicular to CD. draw PG = sine of BC.



Then twice the area of APG. the orthographic projection of the plane triangle ABC upon the plane through A perpendicular to CD, $=\sin a \sin b \sin C$ and likewise the projections of the plane triangle ABC upon planes through B and C, perpendicular to DB and DA are $\sin a \sin c \sin B$, $\sin b \sin c \sin A$, which projections are equal, being upon planes equally inclined to the plane ABC.

Also, $\sin AO \sin BO \sin AOB$ is twice the projection of the plane triangle AOB upon a plane perpendicular to OD, the inclination of which plane to APG is the arc OC. Hence the projection of $\sin AO \sin BO \sin AOB$ upon APG is $\sin AO \sin BO \sin AOB \cos OC$.

Through A, draw AK, the sine of AO, KH perpendicular to CD, and HL parallel and equal to PG; then the triangle LKH is the projection of the plane triangle OBC upon a plane perpendicular to CD, and $LH \times HK \times \sin C = \sin a \cos AO \sin CO \sin C = \sin BO \sin CO \sin BOC \cos AO$.

In like manner the projection of the plane triangle AOC upon a plane perpendicular to CD is equal to $\sin AO \sin CO \sin AOC \cos BO$. Hence $\sin a \sin b \sin C = \sin a \sin c \sin B = \sin b \sin c \sin A = \sin AO \sin BO \times \sin AOB \cos CO + \sin BO \sin CO \sin BOC \cos AO + \sin AO \sin CO \sin AOC \cos BO = \sin AO \sin BO \sin CO (\cot AO \sin BOC + \cot BO \sin AOC + \cot CO \sin AOB)$.

NOTE ON THE SOLUTION OF PROBLEM 260, BY THE EDITOR —In the solution of this problem (pp. 121–22), the equations from Routh were incorrectly written, and should be corrected as follows: In (1), for —, read +, and in (2), for "sin", read cos; also, in (3), for "cos" read sin.

By introducing these corrections in the solution at p. 121 we get

$$\frac{d^2\varphi}{dt^2} = \frac{5g}{7(R-r)}\cos\varphi = \frac{\frac{5}{7}g}{l}\cos\varphi, \qquad (6)$$

where l is put for $R-r$. Integrating once, and reducing, we find

$$dt = \sqrt{\left(\frac{l}{\frac{10}{7}g}\right)} \cdot \frac{d\varphi}{\surd(\sin\varphi)}.$$

If we put $l\sin\varphi = x$, we have

$$d\varphi = \frac{dx}{\surd(l^2-x^2)}; \; \therefore \; dt = \frac{l}{\surd(\frac{10}{7}g)} \cdot \frac{dx}{\surd[x(l^2-x^2)]}.$$

This value of dt is (see Brande's Encyc., art. pend.) the differential of the time of a semi-oscillation of a pendulum whose length is l, through an arc $= \pi$, or a semi-circumference; the accelerating force being $\frac{5}{7}g$.

We are under obligations to Mr. Adcock and Mr. Siverly for calling our attention to the errors above alluded to. The above corrections being introduced, this solution agrees with Mr. Adcock's solution, he writes, "by the principle that twice the work of gravity equals *vis viva* due to translation plus that due to rotation."

"If t' is the time of a simple pendulum through the same arc φ, $t = t'\surd\frac{7}{5}$; length of pendulum $= R-r$."

SOLUTIONS OF PROBLEMS IN NUMBER FOUR.

Solutions of problems in No. 4 have been received as follows:

From R. J. Adcock, 271, 272; G. M. Day, 275; Geo. Eastwood, 267; Prof. Edgar Frisby, 266; H. Heaton, 266, 271; W. E. Heal, 270, 274; Prof. A. Hall, 275; Chas. H. Kummell, 270, 271, 273, 274, 275; Prof. D. J. McAdam, 269, 270, 274, 275; Artemas Martin, 271; P. Richardson, 270; E. B. Seitz, 267, 269, 270, 271, 272, 273, 274, 275.

We have also received from Prof. W. P. Casey, of San Francisco, Cal., a very elaborate and elegant solution of problem 85, and a geometrical construction of problem 126; also a very simple construction of problem 234; all of which we would be pleased to publish if our space permitted.

At the time of the publication of problem 85, we received several solutions of it, but as all the solutions that have yet been received, except for particular cases, involve equations of a higher degree than the second, we think the quadrilateral has not yet been "constructed". Nevertheless, as the solution by Prof. Casey, above alluded to, is perhaps the most complete and elegant that has been received, it will be inserted as soon as our space will permit; and we hope also to be able, at some future time, to present the geometrical construction of 126, above referred to.